

Comment on “Nonuniversality of Transverse Momentum Dependent Parton Distributions at Small x ”

Hsiang-nan Li

Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China

We point out that the analysis in arXiv:1003.0482 actually verifies the universality of transverse momentum dependent quark distributions at small x , which supports the observation in our earlier work arXiv:0904.4150. Once the gluon exchanges responsible for the additional infrared divergences are factorized and summed into an exponential factor, it can be expressed as a matrix element of Wilson line operators, and treated as a nonperturbative input to the k_T factorization of pA collisions. The same exponential factor has been extracted from pp collisions in arXiv:0904.4150, rendering possible experimental constraints on its behavior from pp processes, and predictions for pA processes.

In a recent letter [1], Xiao and Yuan analyzed the initial and final state interaction effects in dijet-correlations in pA collisions at small x , and demonstrated the nonuniversality of the transverse momentum dependent (TMD) quark distributions in a scalar-QED model. The nonuniversality is caused by the infrared divergence attributed to vanishing invariant mass squared $-k_\perp^2$ of a Glauber gluon, k_\perp being the gluon transverse momentum. As a comparison, the all-order amplitudes for the TMD quark distributions (see Eq. (14) and Eq. (39) in [2])

$$\begin{aligned}\tilde{A}_{\text{DIS}} &= iV(r_\perp) \left[1 - e^{-igg_1 W(\mathbf{r}_\perp, \mathbf{R}_\perp)} \right], \\ \tilde{A} &= iV(r_\perp) \left\{ 1 - e^{igg_1 [G(\mathbf{R}_\perp + \mathbf{r}_\perp) - G(\mathbf{R}_\perp)]} \right\} \\ &\quad \times e^{-igg_2 G(\mathbf{R}_\perp)},\end{aligned}\quad (1)$$

were extracted from the deep inelastic lepton-nucleus scattering (DIS) and the dijet-correlation process $p+A \rightarrow \text{Jet}1 + \text{Jet}2 + X$, respectively. The factor $G(\mathbf{R}_\perp + \mathbf{r}_\perp) - G(\mathbf{R}_\perp)$ with $G(\mathbf{R}_\perp) = K_0(\lambda R_\perp)/(2\pi)$, K_0 being the Bessel function, and λ an infrared regulator, approaches $-W(\mathbf{r}_\perp, \mathbf{R}_\perp) = (1/2\pi) \ln(R_\perp/|\mathbf{R}_\perp + \mathbf{r}_\perp|)$ in the $\lambda \rightarrow 0$ limit. The additional exponential factor $e^{-igg_2 G(\mathbf{R}_\perp)}$ arises from the summation of the Glauber divergences to all orders. The TMD quark distribution for the nucleus A from the dijet correlation was then given by

$$\begin{aligned}\tilde{q}(x, q_\perp) &= \frac{xP^{+2}}{8\pi^4} \int dp^- p^- \int d^2 R_\perp d^2 R'_\perp d^2 r_\perp \\ &\quad \times e^{i\mathbf{q}_\perp \cdot (\mathbf{R}_\perp - \mathbf{R}'_\perp)} e^{-igg_2 [G(\mathbf{R}_\perp) - G(\mathbf{R}'_\perp)]} \\ &\quad \times V(r_\perp) \{ 1 - e^{igg_1 [G(\mathbf{R}_\perp + \mathbf{r}_\perp) - G(\mathbf{R}_\perp)]} \} \\ &\quad \times V(r'_\perp) \{ 1 - e^{igg_1 [G(\mathbf{R}'_\perp + \mathbf{r}'_\perp) - G(\mathbf{R}'_\perp)]} \},\end{aligned}\quad (2)$$

with $\mathbf{r}'_\perp = \mathbf{R}_\perp + \mathbf{r}_\perp - \mathbf{R}'_\perp$. It was claimed that the Glauber factor breaks the universality of the TMD quark distribution [1].

We first point out that the same Glauber factor $e^{-iS(\mathbf{b})}$ has been derived in our earlier work on the di-hadron production in pp collisions at small x in the same model [3], with the exponent S from the one-loop correction,

$$S(\mathbf{b}) = \frac{g^2}{(2\pi)^2} \int \frac{d^2 k_\perp}{k_\perp^2 + \lambda^2} e^{-i\mathbf{k}_\perp \cdot \mathbf{b}}. \quad (3)$$

Equation (3) leads to $G(\mathbf{R}_\perp)$ in [1] with the impact parameter \mathbf{b} . The difference is that we did not differentiate the coupling constants g_2 and g associated with the parton lines from the projectile and from the target, respectively. The Feynman rule in Eq. (3) was constructed by means of the eikonal approximation for the Glauber gluons at small x [3]. Hence, it is straightforward to obtain the definition of the Glauber factor

$$\begin{aligned}e^{-igg_2 G(\mathbf{R}_\perp)} &= \langle 0 | W_-(\mathbf{R}_\perp; -\infty)^\dagger W_-(\mathbf{R}_\perp; \infty) \\ &\quad \times W_+(\mathbf{0}; \infty) W_+(\mathbf{0}; -\infty)^\dagger | 0 \rangle,\end{aligned}\quad (4)$$

where the Wilson line operator is written as

$$W_-(\mathbf{R}_\perp; \infty) = P e^{-ig_2 \int_0^\infty dz u_- \cdot A(\mathbf{R}_\perp + z u_-)}. \quad (5)$$

The expression of W_+ is similar, but with the light-cone vector $u_-^\mu = (0, 1, \mathbf{0})$ being replaced by $u_+^\mu = (1, 0, \mathbf{0})$, and g_2 by g . The net effect of $W_-(\mathbf{R}_\perp; -\infty)^\dagger W_-(\mathbf{R}_\perp; \infty)$ and $W_+(\mathbf{0}; \infty) W_+(\mathbf{0}; -\infty)^\dagger$ demands the vanishing of the components k^+ and k^- of the gluon momentum, respectively. It is easy to show, by expanding the Wilson line operators order by order, that Eq. (4) reproduces Eq. (3).

Obviously, Eqs. (1) and (2) indicate that the Glauber divergences are factorizable in the impact parameter space. Once the factorization is achieved, the Glauber factor can be treated as an additional nonperturbative input to pA collisions, just like the soft factor introduced in the k_T factorization [4]. The same Glauber factor was derived in [3] and [1, 2], rendering possible experimental constraints on its behavior from pp processes, and predictions for pA processes. Based on the observations that the Glauber factor can be expressed as a matrix element of the Wilson line operators, treated as a convolution piece in the k_T factorization, and constrained from some processes, we postulate that the analysis in [1] actually verifies the universality of TMD quark distributions at small x as claimed in [3].

-
- [1] B.W. Xiao and F. Yuan, Phys. Rev. Lett. **105**, 062001 (2010) [arXiv:1003.0482].
 - [2] B.W. Xiao and F. Yuan, arXiv:1008.4432.
 - [3] C.P. Chang and H-n. Li, arXiv:0904.4150.
 - [4] J.C. Collins, T.C. Rogers, and A.M. Stasto, Phys. Rev. D **77**, 085009 (2008).